HW 2-2, 2-3, 2-4 (Kairos week caused a gap)

3.1 p 172 41*, 45 - 57 odd, 58, 59

3.2 p 178 3-25 odd, 29

Read 3.3

3.1

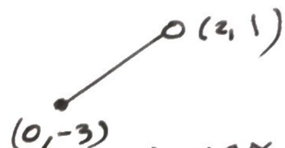
41 $f(x)=2x-3$ $f'(x)=2\neq 0$ find extrema

a) $[0,2]$

x	0	2
$f(x)$	-3	1

Ab. min max (EVT)

b) $[0,2)$



Ab. min, No max

c) $(0,2]$

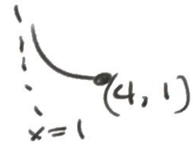
Ab max $(2,1)$
no min

d) $(0,2)$
no min or max

Calc ok

45

$f(x)=\dfrac{3}{x-1}$ $f'(x)=\dfrac{-3}{(x-1)^2}$ ⇒ never 0



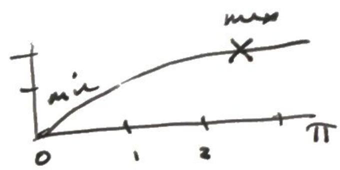
on $(1,4]$? asymptote at 1, $f(4)=1$ so Ab min at 1.

Calc OK

47 $f(x)=\sqrt{x}+\dfrac{\sin x}{3}$ on $[0,\pi]$

using calc > Max occurs (2.7149, 1.7856)

Ab min @ (0,0)



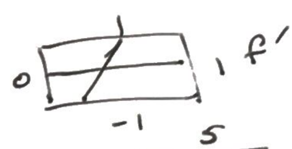
Calc ok - find crit. w/ calculator

49 $f(x)=3.2x^5+5x^3-3.5x$ on $[0,1]$

cont, so EVT ok

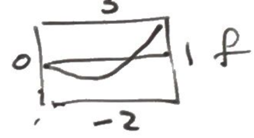
$f'(x)=16x^4+15x^2-3.5$

zero slope at .4398018l → A



x	0	.43980181	1
$f(x)$	0	-1.061307	4.7

Ab min Ab max on $[0,1]$



Calc

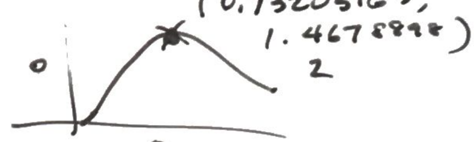
51 find max $|f''(x)|$ on $[0,2]$ if $f(x)=\sqrt{1+x^3}=(1+x^3)^{1/2}$

$f'(x)=\frac{1}{2}(1+x^3)^{-1/2}(3x^2)=\dfrac{3x^2}{2\sqrt{1+x^3}}$

$f''(x)=\dfrac{(6x)(2\sqrt{1+x^3})-3x^2(1+x^3)^{-1/2}(3x^2)}{4(1+x^3)}$

$=\left|\dfrac{12x(1+x^3)-9x^4}{4(1+x^3)\sqrt{1+x^3}}\right|$

$=\left|\dfrac{12x+3x^4}{4(1+x^3)^{3/2}}\right|$



CALC > MAX

HW 2-2 to 2-4 page 2

3.1 53 Extrema of fourth derivative of f:

$$f(x) = (x+1)^{4/3} \quad \text{on } [0,2] \qquad \text{(calc/computer ok)}$$

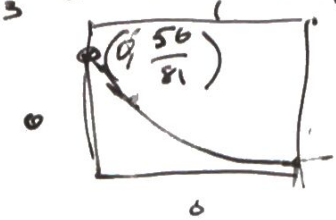
$$f'(x) = \frac{4}{3}(x+1)^{1/3}$$

$$f''(x) = \frac{1}{3} \cdot \frac{4}{3}(x+1)^{-2/3}$$

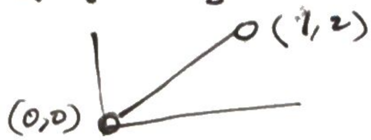
$$f'''(x) = -\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{4}{3}(x+1)^{-5/3}$$

$$f^{(4)}(x) = -\frac{5}{3} \cdot -\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{4}{3}(x+1)^{-8/3} = \frac{40}{81}(x+1)^{-8/3}$$

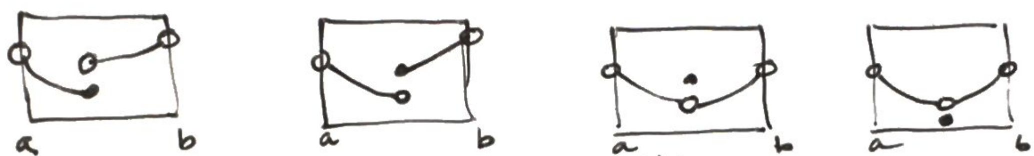
Using calc.



55 a cont. function on an open interval may not have extrema. for example $y = 2x$ on $(0,1)$



57-58 - Extrema on (a, b) given the graph



has Ab min | No extrema | No extrema | Has Ab min

59 Consider $f(x) = \frac{x-4}{x+2}$ Is $x = -2$ a critical number?

on any interval containing $x=-2$, yes.
Why? critical numbers are where the slope is 0 or DNE, here it DNE due to the vertical asymptote at $x=-2$

3.2 p178 3-25 odd, 29

(3) $f(x) = \left|\frac{1}{x}\right|$, on $[-1, 1]$

Rolle's Th doesn't apply, not cont at $x=0$

(5) $f(x) = 1 - |x-1|$ on $[0, 2]$

Rolle Th don't apply, not diff at $x=1$

(7) $f(x) = x^2 - x - 2 = (x+1)(x-2)$

so $f(-1) = f(2) = 0$

$f'(x) = 2x - 1$ when is $2x - 1 = 0$?

$x = \frac{1}{2}$ $f'(\frac{1}{2}) = 0$

$-1 \leq \frac{1}{2} \leq 2$ ✓

(9) $f(x) = x\sqrt{x+4}$

$f(0) = f(-4) = 0$

$f'(x) = \frac{x}{2\sqrt{x+4}} + \sqrt{x+4}$ when does it equal 0 ?

$\sqrt{x+4} = \frac{-x}{2\sqrt{x+4}}$

$2(x+4) = x$

$3x = -8$

$x = -\frac{8}{3}$, $f'(\frac{-8}{3}) = 0$

and $0 < -\frac{8}{3} < -4$

(11) $f(x) = -x^2 + 3x$ on $[0, 3]$

can Rolle be used ? cont. ✓ diff ✓ $f(0) = f(3) = 0$ ✓

find $f'(x) = -2x + 3$, $-2x + 3 = 0$

$x = \frac{3}{2}$

(13) $f(x) = (x-1)(x-2)(x-3)$ on $[1, 3]$

can Rolle be used? cont ✓ diff ✓ $f(1) = f(3) = 0$

$f(x) = x^3 - 6x^2 + 11x - 6$ $f'(x) = 3x^2 - 12x + 11 = 0$ when

$x = 2 \pm \frac{\sqrt{3}}{2}$

(15) $f(x) = x^{2/3} - 1$ on $[-8, 8]$

not diff at $x = 0$

No Rolle's

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{x^{1/3}}$ ← dis v bey 0

(17) $f(x) = \frac{x^2-2x-3}{x+2}$ on $[-1,3]$ $f(-1) = f(3) = 0$

since -2 is avoided, continuous, diff'ble

$f'(x) = \frac{(x+2)(2x-2)-(x^2-2x-3)}{(x+2)^2} = \frac{2x^2+2x-4-x^2+2x+3}{(x+2)^2}$

$= \frac{x^2+4x-1}{(x+2)^2}$ when is $x^2+4x-1=0$?

$(x+2)^2 = 1+4$

$x = -2 \pm \sqrt{5}$

only $-2+\sqrt{5}$ is on $[-1,3]$

(19) $f(x) = \sin x$ on $[0, 2\pi]$

yes on Rolle: cont, diff, $f(0) = f(2\pi) = 0$

$f'(x) = \cos x$, $\cos x = 0$

extrema @ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

(21) $f(x) = \cos \pi x$ on $[0,2]$

yes on Rolle: $f(0) = f(2) = 1$

$f'(x) = -\pi \sin \pi x$ and when $-\pi \sin \pi x = 0$

$\sin \pi x = 0$

$\pi x = 0, \pi, 2\pi$

extrema @ $x = 0, 1, 2$

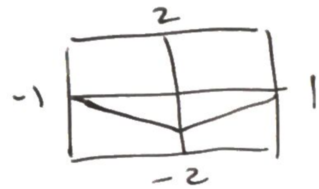
min at $(1,-1)$

(23) $f(x) = \tan x$ on $[0,\pi]$

Not continuous at $x = \frac{\pi}{2}$ so No Rolle's

(25) $f(x) = |x| - 1$ on $[-1, 1]$ with calc.

Not Diff. at $x=0$ so No Rolle's



(29) Hight of ball: $f(t) = -16t^2 + 48t + 6$ feet when t is secs

$f(1) = f(2) = 38$ feet; cont & diff, so by Rolle's Th

some time between 1 and 2 seconds

the hight will switch from going up to

going down ($f'(c) = 0$ for some c in $(1,2)$)

$f'(t) = -32t + 48$; $-32t+48=0$

$t = \frac{-48}{-32} = \frac{3}{2}$ sec.